GROUNDWATER REGIME IN FILTRATION FROM A SPRINKLER IN AN IRRIGATION SYSTEM

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The studies [1, 2] examine the problem of two-dimensional steady-state filtration (in accordance with Darcy's law) in a uniform isotropic soil from a furrow-type sprinkler in the presence of capillarity in the soil. It was assumed in these studies that there was a water absorbing drainage stratum extending a great depth (theoretically to infinity) beneath the surface. The case of a drainage layer of finite depth was examined in [3], where it was noted that evaporation usually has relatively little effect on the filtration characteristics of the flow being considered. Here, we construct a solution to the problem of filtration from a furrow-type sprinkler through a layer of soil on top of a highly permeable bed containing headwater. We study the character of the dependence of the rate of filtration from the sprayer on the capacity of the layer, the diameter of the sprayer, the head, and the caillarity of the soil.

Figure 1 schematically depicts the right half of the region of water movement from the sprinkler through a layer of soil of thickness T into the underlying, highly permeable head layer. The water head H in the head layer is reckoned from the boundary between this layer and the filtration layer. The sprinkler is filled with water to the surface of the ground, which is assumed to be a horizontal plane.

In the initial part of our investigation, we replace the sprinkler with a point source located at point A. We assume that $\psi|_{AF} = 0$. The conditions $\varphi = -y + h_v$, $\psi = Q/2$, where $\omega = \varphi + i\psi$ must be satisfied on the depression curve, where h is the complex flow potential referred to the soil filtration coefficient and Q is the filtration rate from the sprinkler referred to the same coefficient. Then $\varphi = T - H$ along the highly permeable layer. In solving the problem, besides Q, it is of great practical interest to know the radius L of the capillary flow of water away from the sprinkler.

We will conformally map the region of the complex potential ω (Fig. 2) and the region dz/dω (Fig. 3) — the inversion of the rate curve — onto the upper half-plane of the variable ξ (Fig. 4).

In accordance with the Christoffel—Schwartz formula, we find that

$$\omega = T - H - \frac{Q}{\pi} \operatorname{arsh} \sqrt{\frac{a-\xi}{(1-a)\xi}}; \tag{1}$$

$$\frac{dz}{d\omega} = -\frac{2i}{\pi}\left(\operatorname{arctg}\sqrt{\frac{\xi}{1-\xi}} + A\sqrt{\frac{\xi}{1-\xi}}\right). \tag{2}$$

Here

$$a = \operatorname{th}^2 \frac{\pi(T-H-h_v)}{Q}, \quad A = b - 1 \quad (0 \leqslant A < +\infty) \tag{3}$$

[a and b are parameters of the conformal mapping (Fig. 4)].

It should be noted that direct elimination of the parameter ξ from Eqs. (1) and (2) does not make it possible to completely solve the problem. We introduce a new auxiliary variable τ, assuming

$$\xi = \sin^2 \tau. \tag{4}$$

Substitution of (4) changes the half-plane ξ into the half-strip τ (Fig. 5), while differentiation of (1) with respect to τ yields the following with allowance for (4)

$$\frac{d\omega}{d\tau} = \frac{\sqrt{a}Q}{\pi \sin\tau \sqrt{a - \sin^2\tau}}; \tag{5}$$

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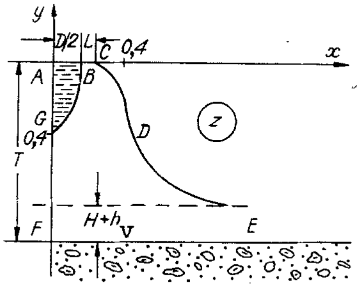

Fig. 1

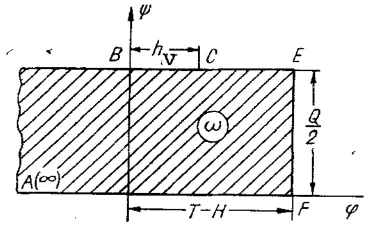

Fig. 2

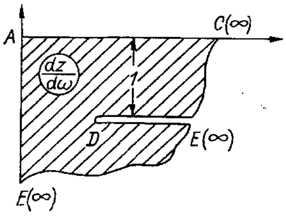

Fig. 3

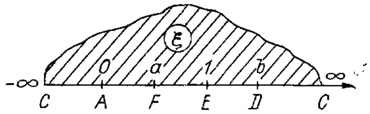

Fig. 4

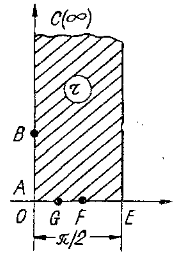

Fig. 5

$$\frac{dz}{d\tau} = -\frac{2i\sqrt{a}Q\,(\tau + A\,\mathrm{tg}\,\tau)}{\pi^2 \sin\tau \sqrt{a - \sin^2\tau}}. \tag{6}$$

Equations (5) and (6) constitute a parametric solution of the problem for the source. Now we apply the results to the case of a sprinkler having a small cross section which is close to being a semicircle. To do this, we take one of the lines of equal head, say BG (see Fig. 1), as a cross section of the flow in a sprinkler with the diameter D. We set $\varphi = 0$ at this section and we use μ to represent the affix of point B in the plane τ. As a result, the theoretical relations will contain four unknown constants a, μ, A, and Q. These quantities will be determined by means of the capacity of the layer T, the head H, the radius of the sprinkler D/2, and the vacuum produced by capillary forces in the soil h_V.

If we integrate (5) and (6) along different sections of the boundary of the region and we eliminate singularities at the ends of certain intervals by a method similar to that used in [4, p. 131], we obtain

$$A = \pi\sqrt{\frac{1-a}{a}}\left[\frac{T}{Q} - \frac{2}{\pi^2}\int\limits_0^{\pi/2} \frac{\arcsin(\sqrt{a}\sin t)\,dt}{\sin t\sqrt{1 - a\sin^2 t}}\right]; \tag{7}$$

$$Q = \pi h_V\left[\mathrm{arth}\,\frac{(\mathrm{ch}\,\mu - \sqrt{a + \mathrm{sh}^2\mu})\sqrt{a}}{\sqrt{a + \mathrm{sh}^2\mu} - a\,\mathrm{ch}\,\mu}\right]^{-1}; \tag{8}$$

$$\frac{D}{2} = \frac{2\sqrt{a}Q}{\pi^2}\int\limits_0^{\mu} \frac{(t + A\,\mathrm{th}\,t)\,dt}{\mathrm{sh}\,t\sqrt{a + \mathrm{sh}^2 t}}. \tag{9}$$

System (3), (7)-(9) determines the sought parameters. We then find the radius of capillary flow in the direction away from the sprinkler

$$L = \frac{8\sqrt{a}Q}{\pi^2}\int\limits_0^1 \frac{t[-\ln t + A(1-t^2)(1+t^2)^{-1}]dt}{(1+t^2)\sqrt{1 + (4a-2)t^2 + t^4}},$$

TABLE 1

Series	D	L	Q	H	L	Q	h_V	L	Q
1	0,3	0,2364	1,1707	0	0,2400	1,4713	0,1	0,0999	1,1042
	0,5	0,2716	1,5692	0,3	0,3175	1,1275	0,5	0,4640	1,4795
	0,7	0,2933	1,9834	0,6	0,7589	0,6745	0,8	1,2669	1,5202
	0,9	0,3077	2,4368	0,65	1,0986	0,5912	0,85	1,6969	1,5207
	1,0	0,3131	2,6832	0,69	2,0348	0,5238	0,89	2,7132	1,5208
2	0,3	0,0996	0,8067	0	0,0976	1,1532	0,1	0,1088	0,9618
	0,5	0,1161	1,1150	0,5	0,1308	0,7571	0,3	0,3175	1,1275
	0,7	0,1267	1,4306	0,8	0,3472	0,3350	0,6	1,1308	1,1821
	0,9	0,1338	1,7731	0,85	0,5748	0,2530	0,65	1,5372	1,1827
	1,0	0,1365	1,9585	0,89	1,3765	0,1859	0,69	2,5532	1,1828

TABLE 2

T	L	Q	T	L	Q
1,0	0,0923	0,9079	3,5	0,1119	0,6894
1,5	0,0980	0,8244	4,0	0,1143	0,6714
2,0	0,1025	0,7735	4,5	0,1165	0,6565
2,5	0,1061	0,7378	5,0	0,1185	0,6438
3,0	0,1092	0,7108			

the coordinates of points on the depression curve

$$x=\frac{D}{2}+L+\frac{8\sqrt{a}Q}{\pi^2}\int_0^t \frac{t\,[-\ln t+A\,(1+t^2)\,(1-t^2)^{-1}]\,dt}{(1+t^2)\sqrt{1+(2-4a)\,t^2+t^4}},$$

$$y=-T+H+h_V+\frac{Q}{\pi}\,\operatorname{arsh}\frac{(1-t^2)\sqrt{a}}{(1+t^2)\sqrt{1-a}}\quad(0\leqslant t\leqslant 1),$$

and the ordinate of point G in the sprinkler flow

$$y_G=-\frac{2\sqrt{a}Q}{\pi^2}\int_0^{\mu_1}\frac{(t+A\,\mathrm{tg}\,t)\,dt}{\sin t\,\sqrt{a-\sin^2 t}}$$

$$\left(\mu_1=\arcsin\sqrt{a\,[1+(1-a)\,\mathrm{sh}^2\pi\,(T-H)/Q]^{-1}}\right).$$

Figure 1 shows the depression curve calculated with T = 1, 0, D = 0.3, h_V = 0.1 and H = 0.1.

We should point out certain cases connected with limiting values of the mapping parameters. With b = 1, we have A = 0. This corresponds to filtration without a head [3]. If along with A = 0, we have a = 1, then Eq. (8) leads to the expression Q = $\pi h_V/\ln\cot\mu$, which coincides with Eq. (15) in [1].

Table 1 shows results of calculations of filtration characteristics to explain the effect of D, H, and h_V on the operation of the sprinkler; it consists of two series, and each series consists of three divisions in which one of the parameters D, H, or h_V is varied (the last two so that H + h_V < T) and the other parameters are fixed with T = 1.0, D = 0.4, H = 0.1, h_V = 0.3 (for series 1) and T = 1.0, D = 0.4, H = 0.3, h_V = 0.1 (for series 2). It is evident that an increase in the sprinkler diameter D by a factor of 3.3 for both series leads to an increase in the radius of the capillary flow L and flow rate Q, respectively, by factors of 1.32-1.36 and 2.29-2.43. At D = 0.3, the values of L and Q in the first series exceed the values of these quantities in the second series by factors of 2.35 and 1.46; the changes in these quantities from series to series for other values of D are roughly the same. With a change in H from 0 to T − h_V − 0.01, L and Q change by factors of 8.5 and 2.8 for the first series and 14 and 6.2 for the second series. Meanwhile, the greatest increase in the radius of capillary flow L is seen at values of H close to T − h_V. Thus, a change in H from 0.85 to 0.89 leads to a 140% increase in L. It is also evident from the second division of Table 1 that the higher the level of groundwater relative to the surface, i.e., the greater H, the lower the rate of filtration [5].

However, the capillarity of the soil has the greatest effect on the width of the irrigated area. It is evident from the last division of Table 1 that L increases by a factor of 27.2 in the first series with an increase in the parameter h_V from 0.1 to 0.89. It should be noted that at $h_V \approx 0$ and $h_V \approx T - H$, the radius of capillary flow exceeds the height of capillary rise h_V. Meanwhile, the largest difference is reached at values of h_V close to $T - H$. Thus, in the case $h_V = 0.89$, $L = 2.7132$ and, thus, $L/h_V = 3.0$. As a result, the substantial value of horizontal absorption noted in [1, 5] — even for low-capillarity soils — is confirmed to exist. Calculations showed that an increase in the head H leads to an even greater spread. For example, in the second series of Table 1 with $h_V = 0.69$, we obtain $L/h_V = 3.7$. As regards the flow rate, it changes by 37 and 27% for the values of h_V shown in the third division of Table 1.

Let us follow the effect of the depth of the high-permeability layer at D = 0.3 and $h_V = 0.1$, fixing the value $T - H - h_V = 0.8$. The results of the calculations are shown in Table 2. It is evident that the capacity of the layer nearly ceases to affect the radius of capillary flow at T > 5. At large values of T, the last two values of L differ from one another by no more than 1.5%. The effect of T on flow rate turns out to be somewhat greater; the latter can be considered negligible at T > 7.

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FINITE RATE OF RADIANT HEAT TRANSFER IN A GRAYBODY IN THE PRESENCE OF HEAT SOURCES (SINKS)

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Different intensive heat-transfer processes which take place in the presence of significant temperature gradients are currently being discussed in the literature. The study of such processes is complicated by the need to make allowance for the variable thermophysical properties of the substance being investigated. This applies in particular to radiant heat transfer. Here, the main characteristic of the substance is the mean free path of the radiation, which depends appreciably on temperature [1].

Radiant heat transfer is described by nonlinear integrodifferential equations in accordance with the nonlocal character of interaction of radiation with a substance [1, 2]. In many important cases, it is sufficient to limit the investigation to a graybody approximation [1], assuming that the absorption coefficient is independent of the spectral composition of the radiation. In the event of planar symmetry, the integrodifferential equation has the following form in dimensionless variables [1, 2] in the presence of heat sources (sinks)

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